

Find the slope of the tangent line to the curve $y = \frac{f(x)}{x^2}$ at the point where $x = 3$ if $f(3) = -4$ and $f'(3) = 2$. SCORE: ____ / 4 PTS

$$\frac{dy}{dx} = \frac{f'(x)x^2 - f(x)(x^2)'}{(x^2)^2} = \frac{x^2 f'(x) - 2x f(x)}{x^4} \quad (2)$$

$$\left. \frac{dy}{dx} \right|_{x=3} = \frac{3^2 \cdot f'(3) - 2(3) \cdot f(3)}{3^4} = \frac{9 \cdot 2 - 6 \cdot (-4)}{81} = \frac{42}{81} = \frac{14}{27} \quad \left(\frac{14}{27} \right)$$

Prove the derivative of $\sec x$ using the definition of the derivative function. Show all steps.

SCORE: ____ / 5 PTS

Do NOT use the quotient rule, nor the known derivatives of any other trigonometric functions.

You may use the value of the two limits proved in lecture without proving them again.

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h} \quad \left(\frac{1}{2}\right) \\ &= \lim_{h \rightarrow 0} \frac{\cos x - \cos(x+h)}{h \cos(x+h) \cos x} \quad \left(\frac{1}{2}\right) \\ &= \lim_{h \rightarrow 0} \frac{\cos x - (\cos x \cos h - \sin x \sin h)}{h \cos(x+h) \cos x} \quad (1) \\ &= \lim_{h \rightarrow 0} \frac{\cos x (1 - \cos h) + \sin x \sin h}{h \cos(x+h) \cos x} \\ &= \lim_{h \rightarrow 0} -\frac{1}{\cos(x+h)} \cdot \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} \quad \left(\frac{1}{2}\right) \\ & \quad + \lim_{h \rightarrow 0} \frac{\sin x}{\cos(x+h) \cos x} \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \quad \left(\frac{1}{2}\right) \\ &= -\frac{1}{\cos x} \cdot 0 + \frac{\sin x}{\cos^2 x} \cdot 1 \quad \left(\frac{1}{2}\right) \\ &= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \tan x \quad \left(\frac{1}{2}\right) \end{aligned}$$

The position of an object at time t is given by $s(t) = (t^3 - 5t)3^t$.

SCORE: ____ / 5 PTS

Find the **acceleration** of the object at time $t = 2$.

$$s'(t) = (t^3 - 5t)' 3^t + (t^3 - 5t)(3^t)' = \underbrace{(3t^2 - 5)3^t}_{\textcircled{1}} + \underbrace{(t^3 - 5t)3^t \ln 3}_{\textcircled{1}}$$

$$s''(t) = (3t^2 - 5)' 3^t + (3t^2 - 5)(3^t)' + (t^3 - 5t)' 3^t \ln 3 + (t^3 - 5t)(3^t \ln 3)'$$
$$= \underbrace{6t \cdot 3^t}_{\textcircled{\frac{1}{2}}} + \underbrace{(3t^2 - 5)(3^t \ln 3)}_{\textcircled{\frac{1}{2}}} + \underbrace{(3t^2 - 5)(3^t \ln 3)}_{\textcircled{\frac{1}{2}}} + \underbrace{(t^3 - 5t)(3^t (\ln 3)^2)}_{\textcircled{\frac{1}{2}}}$$

$$s''(2) = 12 \cdot 9^{\textcircled{\frac{1}{2}}} + 7 \cdot 9 \ln 3 + 7 \cdot 9 \ln 3 + (-2) \cdot 9 (\ln 3)^2$$

$$= \underbrace{108 + 126 \ln 3 - 18 (\ln 3)^2}_{\textcircled{1}}$$

Prove the derivative of $\cot x$ using the quotient rule. Show all steps.

SCORE: ____ / 3 PTS

You may use the known derivatives of $\sin x$ and $\cos x$ without proving them.

$$\cot x = \frac{\cos x}{\sin x}$$

$$(\cot x)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{(\sin x)^2}$$

$$= \frac{(-\sin x) \sin x - \cos x (\cos x)}{(\sin x)^2} \quad \left(\frac{1}{2}\right)$$

$$\frac{-(\sin^2 x + \cos^2 x)}{(\sin x)^2} \quad \left(\frac{1}{2}\right)$$

$$= \frac{-1}{\sin^2 x}$$

$$= \left[-\frac{1}{\sin^2 x} \right] \quad \left(\frac{1}{2}\right)$$

$$= \left[-\csc^2 x \right] \quad \left(\frac{1}{2}\right)$$

$$\frac{d}{dt}(\csc t + \tan t \sec t)$$

$$= -\csc t \cot t + \sec^2 t \sec t$$

$$+ \tan t \sec t \tan t$$

$$= \underline{-\csc t \cot t} + \underline{\sec^3 t}, \textcircled{1}$$

$$\textcircled{1}$$

$$+ \underline{\sec t \tan^2 t}, \textcircled{1}$$

$$\frac{d}{dy}(5y^e + 7e^y - 3\pi^e)$$

$$= \underbrace{5ey^{e-1}}_{\textcircled{1}} + \underbrace{7e^y}_{\textcircled{1}}$$

+ $\textcircled{\frac{1}{2}}$ FOR NOT WRITING

A DERIVATIVE FOR $-3\pi^e$
(IE, DERIVATIVE = 0)

$$\frac{d}{dx} \frac{2-3x^2+x^4}{1-2x^4} \quad (\text{Your final answer must be one fraction})$$

$$= \frac{(-6x+4x^3)(1-2x^4) - (2-3x^2+x^4)(-8x^3)}{(1-2x^4)^2} \quad \textcircled{\frac{3-1}{2}}$$

$$\frac{(-6x+4x^3)(1-2x^4) - (2-3x^2+x^4)(-8x^3)}{(1-2x^4)^2} \quad \textcircled{\frac{1}{2}}$$

$$= \frac{-6x + 12x^5 + 4x^3 - 8x^7 + 16x^3 - 24x^5 + 8x^7}{(1-2x^4)^2}$$

$$= \frac{-12x^5 + 20x^3 - 6x}{(1-2x^4)^2} \quad \textcircled{1}$$

$$\frac{d^3}{dr^3} \frac{18r^2 - 27r}{4\sqrt[3]{r}}$$

$$= \frac{d^3}{dr^3} \left(\frac{9}{2} r^{\frac{5}{3}} - \frac{27}{4} r^{\frac{2}{3}} \right)$$

$$= \frac{d^2}{dr^2} \left(\frac{15}{2} r^{\frac{2}{3}} - \frac{9}{2} r^{-\frac{1}{3}} \right) \textcircled{1\frac{1}{2}}$$

$$= \frac{d}{dr} \left(5r^{-\frac{1}{3}} + \frac{3}{2} r^{-\frac{4}{3}} \right) \textcircled{1}$$

$$= \left[-\frac{5}{3} r^{-\frac{4}{3}} - 2r^{-\frac{7}{3}} \right] \textcircled{1}$$